P.R.Government College (Autonomous), Kakinada III year B.Sc., Degree Examinations- V Semester (w.e.f 2016-17) Paper III (Advanced) Core Course: Linear algebra

Total Hrs. of Teaching-Learning: 45 @ 3h / Week

Total Credits: 03

Objective:

 To improve the students ability of understanding the most application oriented topic in Mathematics that is Linear Algebra.

• To equip the skill of understanding the concepts and writing the proofs of the

Theorems.

Module-I: Vector Space and Linear Transformations

Unit 1: Vector spaces-Definition, Subspace, Algebra subspaces, Linear combination of vectors, Basis and Dimension, Linear sum of subspaces, Quotient spaces. (14 hrs)

Unit 2: Linear transformations – Definition, Rangespace and NullSpace, Algebra of linear transformations, Linear transformations-Matrices. (9 hrs)

Module-II: Characteristic Values and Characteristic Vectors and Inner Product spaces

Unit 3: Characteristic Values and Characteristic Vectors- Cayley-Hamilton Theorem (for matrices) (9 hrs)

Unit 4: Inner Product spaces – Definitions and examples ,orthogonality and orthonormality, Gram-Schmidt Orthogonalisation Process. (13 hrs)

Prescribed Text Books:

J.N. Sharma & A.R. Vasista, Linear Agebra, Krishna Prakasham Mandir, Meerut.

Books for Reference:

- 1. III year Mathematics Linear Algebra and Vector Calculus, Telugu Academy.
- 2. A Text Book of B.Sc. Mathematics Vol III, S.Chand&Co.

QUESTION PAPER PATTERN, SEMESTER-V, PAPER -III CORE

Module	TOPIC	V.S.A.Q	S.A.Q (including choice)	E.Q (including choice)	Marks Allotted
Module-I	Vector Space	02	02	02	28
	Linear Transformations	02	03	01	25
Module-II	Char. values and char. vectors	02	02	01	20
	Inner product spaces	02	03	02	33
Total		08	10	06	

V.S.A.Q. = Very Short answer questions (1mark)
S.A.Q. = Short answer questions (5 marks)
E.Q. = Essay questions (8 marks)

Very Short answer questions $: 8 \times 1M = 08$ Short answer questions $: 6 \times 5M = 30$ Essay questions $: 4 \times 8M = 32$

Total Marks 70

P.R GOVT.COLLEGE (AUTONOMOUS), KAKINADA III B.Sc. EXAMINATION, MATHEMATICS MODEL PAPER – V SEMESTER, PAPER-III

Core (Advanced): Linear Algebra

Max.Marks: 70M Time: 3 hours PART -1

Answer the following questions. Each question carries 1 mark.

8x1M =8M

- If S. T are the subspaces of vector space V(F) then L(SUT)=
- 2. The standard basis of $V_2(R)$ is.....
- Define range space.
- 4. Find the null space of the transformation $T: \mathbb{R}^2 \to \mathbb{R}^3$ defined by T(x,y) = (x+y, x-y, y)
- Define Eigen vector of a square matrix.
- 6. Find the Eigen values of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$
- 7. Find the length of the vector $\alpha = (2,1,1+i)$
- Write Bessel's inequality.

PART -II

Answer any Six questions by choosing Three from each section. SECTION - A

 $6 \times 5M = 30M$

- 9. Determine whether the following set of vector is L.D or L.I {(1,-2,1),(2,1,-1)(7,-4,1)}.
- 10. Show that the set of vectors $\{(2,1,4),(1,-1,2),(3,1,-2)\}$ form a basis for \mathbb{R}^3 .
- 11. Find T(x, y, z) where $T: R^3 \to R$ is defined by T(1,1,1)=3, T(0,1,-2)=1, T(0,0,1)=-2.
- 12. Let T: $V_4 \rightarrow V_3$ be a linear transformation defined by $T(\alpha_1) = (1,1,1)$; $T(\alpha_2) = (1,-1,1)$; $T(\alpha_3)=(1,0,-1)$ then verify that $\rho(T)+\nu(T)=\dim V_4$
- State and prove rank and nullity theorem.

SECTION - B

- State and prove Cayley-Hamilton theorem.
- 15. If $A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$, test A for diagonalizability.
- 17. Prove that $\{(3/5,0,4/5),(-4/5,0,3/5),(0,1,0)\}$ form an orthogonal subset of R3(R) space.

18. State and prove Parsevel's identity.

PART -III

Answer any Four questions by choosing at least ONE from each section.

4 X 8=32M

SECTION - C

- 19. Let V(F)be a vector space and $S = \{\alpha_1, \alpha_2, ... \alpha_n\}$ is a finite subset of non-zero vectors of V(F). Then S is linearly dependent if and only if some vector $\alpha_k \in S$, $2 \le k \le n$, can be expressed as a linear combination of its preceding vectors.
- 20. Let W be a sub space of a finite dimensional vector space V(F), then prove that dimV/W=dimV-dimW.
- 21. Find the null space, range, rank and nullity of the transformation T: $R^2 o R^3$ defined by T(x,y) = (x+y,x-y,y).

SECTION - D

22. Find the characteristic roots and the corresponding vectors of the matrix

$$A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$$

- 23. State and Prove Cauchy-Schwarz's inequality.
- 24. Applying Gram Schmidt process obtain an orthonormal basis of $R^3(R)$ from the basis $\{(2,0,1), (3,-1,5), (0,4,2)\}.$

P.R.GOVT.COLLEGE (AUTONOMOUS), KAKINADA III B.Sc. MATHEMATICS, Semester V (w.e.f 2016-2017)

Course Code: Linear Algebra

Total Hrs. of Exercises: 45 hrs @ 3 hr/Week in 15 Sessions

Suggested topics for Problem Solving Sessions

- 1. Vector Spaces
- 2. Basis and Dimensions-I
- 3. Basis and Dimensions-1I
- 4. Linear Transformation
- 5. Characteristic Values and Cayley Hamilton Theorem
- 6. Diagnolizability
- 7. Inner Product Spaces
- 8. Orthogonality

PRACTICAL EXAMINATIONS PATTERN

End of the V semester

(Course: Linear Algebra)

PRACTICAL EXAMINATION FOR V semester: 50 Marks

Written examination: 30 M

Record : 10 M

Cont. Ass. : <u>10 M</u>

TOTAL : <u>50 M</u>